Name:

Spring 2018 Math 245 Exam 2

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 12:40 and will end at 1:30; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. Proof by Contradiction theorem

b. Nonconstructive Existence Proof theorem

c. Existence and Uniqueness Proof theorem

d. Fibonacci numbers

Problem 2. Carefully define the following terms:

a. Proof by (basic) Induction

b. Proof by Reindexed Induction

c. big Omega (Ω)

d. big Theta (Θ)

Problem 3. Suppose that an algorithm has runtime specified by recurrence relation $T_n = 4T_{n/2} + n^2$. Determine what, if anything, the Master Theorem tells us.

Problem 4. Let $x \in \mathbb{R}$. Prove that $\lceil x \rceil$ is unique; that is, prove that there is at most one $n \in \mathbb{Z}$ with $n - 1 < x \leq n$.

Problem 5. Let $x \in \mathbb{R}$. Prove that $\lceil x \rceil$ exists; that is, prove that there is at least one $n \in \mathbb{Z}$ with $n - 1 < x \leq n$.

Problem 6. Let $n \in \mathbb{Z}$. Prove that $\frac{(n-1)n(n+1)}{3} \in \mathbb{Z}$.

Problem 7. Solve the recurrence given by $a_0 = 0, a_1 = 6$, $a_n = a_{n-1} + 2a_{n-2}$ (for $n \geq 2$).

Problem 8. Let $r \in \mathbb{R}$. Use induction to prove that for all $n \in \mathbb{N}_0$, $(1-r)\sum_{i=0}^{n} r^i = 1 - r^{n+1}$.

Problem 9. Without using the Classification Theorem, prove that $a_n = O(4^n)$, for $a_n = 3^n$. Hint: induction.

Problem 10. Without using the Classification Theorem, prove that $a_n \neq O(2^n)$, for $a_n = 3^n$.